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DOUBLE SEMIOPEN SETS ON DOUBLE BITOPOLOGICAL SPACES

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Abstract. We introduce the concepts of double bitopological spaces as a generalization of intuitionistic fuzzy topological spaces in Šostak's sense and Kandil's fuzzy bitopological spaces. Also we introduce the concept of $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-semiopen sets and double pairwise $(r,s)(u,v)$-semicontinuous mappings in double bitopological spaces and investigate some of their characteristic properties.

1. Introduction

Chang [2] defined fuzzy topological spaces. These spaces and its generalizations are later studied by several authors, one of which, developed by Šostak [12], used the idea of degree of openness. This type of generalization of fuzzy topological spaces was later rephrased by Chattopadhyay, Hazra, and Samanta [3], and by Ramadan [11].

As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov [1]. Çoker and his colleagues [4, 6, 7] introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets. Using the idea of degree of openness and degree of nonopenness, Çoker and M. Demirci [5] defined intuitionistic fuzzy topological spaces in Šostak's sense as a generalization of smooth fuzzy topological spaces and intuitionistic fuzzy topological spaces.

Kandil [8] introduced and studied the notion of fuzzy bitopological spaces as a natural generalization of fuzzy topological spaces.

In this paper, we introduce the concepts of double bitopological spaces as a generalization of intuitionistic fuzzy topological spaces in Šostak's

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sense and Kandil's fuzzy bitopological spaces. We also introduce the concept of $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-semiopen ($(r,s)(u,v)$-semiclosed) sets and double pairwise $(r,s)(u,v)$-semicontinuous ($(r,s)(u,v)$-semiopen and $(r,s)(u,v)$-semiclosed, respectively) mappings in double bitopological spaces and investigate some of their characteristic properties.

2. Preliminaries

Let I be the unit interval $[0,1]$ of the real line. A member μ of I^X is called a fuzzy set of X. For any $\mu \in I^X$, μ^c denotes the complement $1-\mu$. By $\tilde{0}$ and $\tilde{1}$ we denote constant maps on X with value 0 and 1, respectively. All other notations are standard notations of fuzzy set theory.

Let X be a nonempty set. An *intuitionistic fuzzy set* A is an ordered pair

$$A = (\mu_A, \gamma_A)$$

where the functions $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership and the degree of nonmembership, respectively, and $\mu_A + \gamma_A \leq \tilde{1}$.

Obviously every fuzzy set μ on X is an intuitionistic fuzzy set of the form $(\mu, \tilde{1}-\mu)$.

Definition 2.1. [1] *Let A and B be intuitionistic fuzzy sets on X. Then*

(1) $A \subseteq B$ *iff* $\mu_A \leq \mu_B$ *and* $\gamma_A \geq \gamma_B$.
(2) $A = B$ *iff* $A \subseteq B$ *and* $B \subseteq A$.
(3) $A^c = (\gamma_A, \mu_A)$.
(4) $A \cap B = (\mu_A \wedge \mu_B, \gamma_A \vee \gamma_B)$.
(5) $A \cup B = (\mu_A \vee \mu_B, \gamma_A \wedge \gamma_B)$.
(6) $0_\sim = (\tilde{0}, \tilde{1})$ *and* $1_\sim = (\tilde{1}, \tilde{0})$.

Let f be a mapping from a set X to a set Y. Let $A = (\mu_A, \gamma_A)$ be an intuitionistic fuzzy set of X and $B = (\mu_B, \gamma_B)$ an intuitionistic fuzzy set of Y. Then:

(1) The image of A under f, denoted by $f(A)$, is an intuitionistic fuzzy set in Y defined by

$$f(A) = (f(\mu_A), \tilde{1} - f(\tilde{1} - \gamma_A)).$$

(2) The inverse image of B under f, denoted by $f^{-1}(B)$, is an intuitionistic fuzzy set in X defined by

$$f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B)).$$

A *smooth fuzzy topology* on X is a map $T : I^X \to I$ which satisfies the following properties:

(1) $T(\tilde{0}) = T(\tilde{1}) = 1$.
(2) $T(\mu_1 \wedge \mu_2) \geq T(\mu_1) \wedge T(\mu_2)$.
(3) $T(\bigvee \mu_i) \geq \bigwedge T(\mu_i)$.

The pair (X, T) is called a *smooth fuzzy topological space.*

An *intuitionistic fuzzy topology* on X is a family T of intuitionistic fuzzy sets in X which satisfies the following properties:

(1) $0_\sim, 1_\sim \in T$.
(2) If $A_1, A_2 \in T$, then $A_1 \cap A_2 \in T$.
(3) If $A_i \in T$ for all i, then $\bigcup A_i \in T$.

The pair (X, T) is called an *intuitionistic fuzzy topological space.*

Let $I(X)$ be a family of all intuitionistic fuzzy sets of X and let $I \otimes I$ be the set of the pair (r, s) such that $r, s \in I$ and $r + s \leq 1$.

Definition 2.2. [12] *Let X be a nonempty set. An intuitionistic fuzzy topology in Šostak's sense $\mathcal{T}^{\mu\gamma} = (\mathcal{T}^\mu, \mathcal{T}^\gamma)$ on X is a mapping $\mathcal{T}^{\mu\gamma} : I(X) \to I \otimes I (\mathcal{T}^\mu, \mathcal{T}^\gamma : I(X) \to I)$ which satisfies the following properties:*

(1) $\mathcal{T}^\mu(0_\sim) = \mathcal{T}^\mu(1_\sim) = 1$ *and* $\mathcal{T}^\gamma(0_\sim) = \mathcal{T}^\gamma(1_\sim) = 0$.
(2) $\mathcal{T}^\mu(A \cap B) \geq \mathcal{T}^\mu(A) \wedge \mathcal{T}^\mu(B)$ *and* $\mathcal{T}^\gamma(A \cap B) \leq \mathcal{T}^\gamma(A) \vee \mathcal{T}^\gamma(B)$.
(3) $\mathcal{T}^\mu(\bigcup A_i) \geq \bigwedge \mathcal{T}^\mu(A_i)$ *and* $\mathcal{T}^\gamma(\bigcup A_i) \leq \bigvee \mathcal{T}^\gamma(A_i)$.

The $(X, \mathcal{T}^{\mu\gamma}) = (X, \mathcal{T}^\mu, \mathcal{T}^\gamma)$ is said to be an intuitionistic fuzzy topological space in Šostak's sense. Also, we call $\mathcal{T}^\mu(A)$ a gradation of openness of A and $\mathcal{T}^\gamma(A)$ a gradation of nonopenness of A.

Definition 2.3. [10] *Let A be an intuitionistic fuzzy set in an intuitionistic fuzzy topological space in Šostak's sense $(X, \mathcal{T}^\mu, \mathcal{T}^\gamma)$ and $(r, s) \in I \otimes I$. Then A is said to be*

(1) *a $\mathcal{T}^{\mu\gamma}$-fuzzy (r, s)-open set if $\mathcal{T}^\mu(A) \geq r$ and $\mathcal{T}^\gamma(A) \leq s$,*
(2) *a $\mathcal{T}^{\mu\gamma}$-fuzzy (r, s)-closed set if $\mathcal{T}^\mu(A^c) \geq r$ and $\mathcal{T}^\gamma(A^c) \leq s$.*

Let $(X, \mathcal{T}^{\mu}, \mathcal{T}^{\gamma})$ be an intuitionistic fuzzy topological space in Šostak's sense. For each $(r,s) \in I \otimes I$ and for each $A \in I(X)$, the $\mathcal{T}^{\mu\gamma}$*-fuzzy* (r,s)*-closure* is defined by

$$\mathcal{T}^{\mu\gamma}\text{-cl}(A,r,s) = \bigcap\{B \in I(X) \mid A \subseteq B, B \text{ is } \mathcal{T}^{\mu\gamma}\text{-fuzzy } (r,s)\text{-closed}\}$$

and the $\mathcal{T}^{\mu\gamma}$*-fuzzy* (r,s)*-interior* is defined by

$$\mathcal{T}^{\mu\gamma}\text{-int}(A,r,s) = \bigcup\{B \in I(X) \mid A \supseteq B, B \text{ is } \mathcal{T}^{\mu\gamma}\text{-fuzzy } (r,s)\text{-open}\}.$$

LEMMA 2.4. [10] *For an intuitionistic fuzzy set A in an intuitionistic fuzzy topological space in Šostak's sense $(X, \mathcal{T}^{\mu}, \mathcal{T}^{\gamma})$ and $(r,s) \in I \otimes I$, we have:*

(1) $\mathcal{T}^{\mu\gamma}$*-cl*$(A,r,s)^c = \mathcal{T}^{\mu\gamma}$*-int*$(A^c,r,s)$.
(2) $\mathcal{T}^{\mu\gamma}$*-int*$(A,r,s)^c = \mathcal{T}^{\mu\gamma}$*-cl*$(A^c,r,s)$.

A system $(X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ consisting of a set X with two intuitionistic fuzzy topologies in Šostak's sense $\mathcal{T}^{\mu\gamma}$ and $\mathcal{U}^{\mu\gamma}$ on X is called a *double bitopological space*.

Let $(X, \mathcal{T}^{\mu\gamma})$ be an intuitionistic fuzzy topological space in Šostak's sense. Then it is easy to see that for each $(r,s) \in I \otimes I$, the family $(\mathcal{T}^{\mu\gamma})_{(r,s)}$ defined by

$$(\mathcal{T}^{\mu\gamma})_{(r,s)} = \{A \in I(X) \mid \mathcal{T}^{\mu}(A) \geq r \text{ and } \mathcal{T}^{\gamma}(A) \leq s\}$$

is an intuitionistic fuzzy topology on X.

Let (X, T) be an intuitionistic fuzzy topological space and $(r,s) \in I \otimes I$. Then the map $T^{\mu\gamma}_{(r,s)} : I(X) \to I \otimes I$ defined by

$$T^{\mu\gamma}_{(r,s)}(A) = \begin{cases} (1,0) & \text{if } A = 0_{\sim}, 1_{\sim} \\ (r,s) & \text{if } A \in T - \{0_{\sim}, 1_{\sim}\} \\ (0,1) & \text{otherwise} \end{cases}$$

becomes an intuitionistic fuzzy topology in Šostak's sense on X.

Hence, we have that if $(X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ is a double bitopological space and $(r,s),(u,v) \in I \otimes I$, then $(X, (\mathcal{T}^{\mu\gamma})_{(r,s)}, (\mathcal{U}^{\mu\gamma})_{(u,v)})$ is an intuitionistic fuzzy bitopological space. Also, if (X, T, U) is an intuitionistic fuzzy bitopological space and $(r,s),(u,v) \in I \otimes I$, then $(X, (T)^{\mu\gamma}_{(r,s)}, (U)^{\mu\gamma}_{(u,v)})$ is a double bitopological space.

3. $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-semiopen sets

Definition 3.1. *Let A be an intuitionistic fuzzy set of a double bitopological space $(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$ and $(r,s),(u,v)\in I\otimes I$. Then A is said to be*

(1) *a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-semiopen set if there is a $\mathcal{T}^{\mu\gamma}$-fuzzy (r,s)-open set B in X such that $B\subseteq A\subseteq\mathcal{U}^{\mu\gamma}$-cl$(B,u,v)$,*
(2) *a $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-semiopen set if there is a $\mathcal{U}^{\mu\gamma}$-fuzzy (u,v)-open set B in X such that $B\subseteq A\subseteq\mathcal{T}^{\mu\gamma}$-cl$(B,r,s)$,*
(3) *a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-semiclosed set if there is a $\mathcal{T}^{\mu\gamma}$-fuzzy (r,s)-closed set B in X such that $\mathcal{U}^{\mu\gamma}$-int$(B,u,v)\subseteq A\subseteq B$.*
(4) *a $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-semiclosed set if there is a $\mathcal{U}^{\mu\gamma}$-fuzzy (u,v)-closed set B in X such that $\mathcal{T}^{\mu\gamma}$-int$(B,r,s)\subseteq A\subseteq B$.*

Theorem 3.2. *Let A be an intuitionistic fuzzy set of a double bitopological space $(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$ and $(r,s),(u,v)\in I\otimes I$. Then the following statements are equivalent:*

(1) *A is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-semiopen set.*
(2) *A^c is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-semiclosed set.*
(3) *$\mathcal{U}^{\mu\gamma}$-cl$(\mathcal{T}^{\mu\gamma}$-int$(A,r,s),u,v)\supseteq A$.*
(4) *$\mathcal{U}^{\mu\gamma}$-int$(\mathcal{T}^{\mu\gamma}$-cl$(A^c,r,s),u,v)\subseteq A^c$.*

Proof. (1) ⇒ (3) Let A be a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-semiopen set of X. Then there is a $\mathcal{T}^{\mu\gamma}$-fuzzy (r,s)-open set B in X such that $B\subseteq A\subseteq\mathcal{U}^{\mu\gamma}$-cl$(B,u,v)$. Since $B\subseteq A$, we have

$$B=\mathcal{T}^{\mu\gamma}\text{-int}(B,r,s)\subseteq\mathcal{T}^{\mu\gamma}\text{-int}(A,r,s).$$

Hence $A\subseteq\mathcal{U}^{\mu\gamma}$-cl$(B,u,v)\subseteq\mathcal{U}^{\mu\gamma}$-cl$(\mathcal{T}^{\mu\gamma}$-int$(A,r,s),u,v)$.

(3) ⇒ (1) Let $\mathcal{U}^{\mu\gamma}$-cl$(\mathcal{T}^{\mu\gamma}$-int$(A,r,s),u,v)\supseteq A$. Suppose that $B=\mathcal{T}^{\mu\gamma}$-int$(A,r,s)$. Then B is a $\mathcal{T}^{\mu\gamma}$-fuzzy (r,s)-open set and

$$\begin{aligned}B=\mathcal{T}^{\mu\gamma}\text{-int}(A,r,s)\subseteq A&\subseteq\mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(A,r,s),u,v)\\&=\mathcal{U}^{\mu\gamma}\text{-cl}(B,u,v).\end{aligned}$$

Thus A is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-semiopen set.

(1) ⇔ (2) and (3) ⇔ (4) follow from Lemma 2.4. □

Corollary 3.3. *Let A be an intuitionistic fuzzy set of a double bitopological space $(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$ and $(r,s),(u,v)\in I\otimes I$. Then the following statements are equivalent:*

(1) *A is a $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-semiopen set.*
(2) *A^c is a $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-semiclosed set.*
(3) *$\mathcal{T}^{\mu\gamma}$-cl$(\mathcal{U}^{\mu\gamma}$-int$(A,u,v),r,s)\supseteq A$.*

(4) $\mathcal{T}^{\mu\gamma}$-*int*$(\mathcal{U}^{\mu\gamma}$-*cl*$(A^c, u, v), r, s) \subseteq A^c$.

THEOREM 3.4. *Let* $(X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ *be a double fuzzy topological space and* $(r, s), (u, v) \in I \otimes I$.

(1) *Suppose that* A *is a* $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$*-double* $(r, s)(u, v)$*-semiopen set. If* $\mathcal{T}^{\mu\gamma}$*-int*$(A, r, s) \subseteq B \subseteq \mathcal{U}^{\mu\gamma}$*-cl*$(A, u, v)$*, then* B *is a* $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$*-double* $(r, s)(u, v)$*-semiopen set.*
(2) *Suppose that* A *is a* $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$*-double* $(u, v)(r, s)$*-semiopen set. If* $\mathcal{U}^{\mu\gamma}$*-int*$(A, u, v) \subseteq B \subseteq \mathcal{T}^{\mu\gamma}$*-cl*$(A, r, s)$*, then* B *is a* $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$*-double* $(u, v)(r, s)$*-semiopen set.*
(3) *Suppose that* A *is a* $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$*-double* $(r, s)(u, v)$*-semiclosed set. If* $\mathcal{U}^{\mu\gamma}$*-int*$(A, u, v) \subseteq B \subseteq \mathcal{T}^{\mu\gamma}$*-cl*$(A, r, s)$*, then* B *is a* $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$*-double* $(r, s)(u, v)$*-semiclosed set.*
(4) *Suppose that* A *is a* $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$*-double* $(u, v)(r, s)$*-semiclosed set. If* $\mathcal{T}^{\mu\gamma}$*-int*$(A, r, s) \subseteq B \subseteq \mathcal{U}^{\mu\gamma}$*-cl*$(A, u, v)$*, then* B *is a* $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$*-double* $(u, v)(r, s)$*-semiclosed set.*

Proof. (1) Let A be a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$-double $(r, s)(u, v)$-semiopen set and $\mathcal{T}^{\mu\gamma}\text{-int}(A, r, s) \subseteq B \subseteq \mathcal{U}^{\mu\gamma}\text{-cl}(A, u, v)$. Then there is a $\mathcal{T}^{\mu\gamma}$-fuzzy (r, s)-open set C such that $C \subseteq A \subseteq \mathcal{U}^{\mu\gamma}\text{-cl}(C, u, v)$. It follows that

$$\begin{aligned} C = \mathcal{T}^{\mu\gamma}\text{-int}(C, r, s) &\subseteq \mathcal{T}^{\mu\gamma}\text{-int}(A, r, s) \\ &\subseteq B \\ &\subseteq \mathcal{U}^{\mu\gamma}\text{-cl}(A, u, v) \\ &\subseteq \mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-cl}(C, u, v), u, v) \\ &= \mathcal{U}^{\mu\gamma}\text{-cl}(C, u, v). \end{aligned}$$

Hence C is a $\mathcal{T}^{\mu\gamma}$-fuzzy (r, s)-open set and $C \subseteq B \subseteq \mathcal{U}^{\mu\gamma}\text{-cl}(C, u, v)$. Thus B is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$-double $(r, s)(u, v)$-semiopen set.

(2) Similar to (1)

(3) Suppose that A be a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$-double $(r, s)(u, v)$-semiclosed set and $\mathcal{U}^{\mu\gamma}\text{-int}(A, u, v) \subseteq B \subseteq \mathcal{T}^{\mu\gamma}\text{-cl}(A, r, s)$. Then there is a $\mathcal{T}^{\mu\gamma}$-fuzzy (r, s)-closed set C such that $\mathcal{U}^{\mu\gamma}\text{-int}(C, u, v) \subseteq A \subseteq C$. It follows that

$$\begin{aligned} \mathcal{U}^{\mu\gamma}\text{-int}(C, u, v) &= \mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-int}(C, u, v), u, v) \\ &\subseteq \mathcal{U}^{\mu\gamma}\text{-int}(A, u, v) \\ &\subseteq B \\ &\subseteq \mathcal{T}^{\mu\gamma}\text{-cl}(A, r, s) \\ &\subseteq \mathcal{T}^{\mu\gamma}\text{-cl}(C, r, s) \\ &= C. \end{aligned}$$

Hence C is a $\mathcal{T}^{\mu\gamma}$-fuzzy (r,s)-closed set and $\mathcal{U}^{\mu\gamma}\text{-int}(C,u,v) \subseteq B \subseteq C$. Thus B is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-semiclosed set.

(4) Similar to (3) □

It is obvious that every $\mathcal{T}^{\mu\gamma}$-fuzzy (r,s)-open ((r,s)-closed) set is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-semiopen ($(r,s)(u,v)$-semiclosed) set and every $\mathcal{U}^{\mu\gamma}$-fuzzy (u,v)-open ((u,v)-closed) set is a $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-semiopen ($(u,v)(r,s)$-semiclosed) set but the converses need not be true which is shown by the following example.

Example 3.5. *Let $X = \{x,y\}$ and let A_1, A_2, A_3 and A_4 be intuitionistic fuzzy sets of X defined as*

$$A_1(x) = (0.0, 0.7), \quad A_1(y) = (0.4, 0.3);$$

$$A_2(x) = (0.5, 0.2), \quad A_2(y) = (0.6, 0.1);$$

$$A_3(x) = (0.1, 0.4), \quad A_3(y) = (0.7, 0.1);$$

and

$$A_4(x) = (0.6, 0.1), \quad A_4(y) = (0.8, 0.0).$$

Define $\mathcal{T}^{\mu\gamma} : I(X) \to I \otimes I$ and $\mathcal{U}^{\mu\gamma} : I(X) \to I \otimes I$ by

$$\mathcal{T}^{\mu\gamma}(A) = (\mathcal{T}^{\mu}(A), \mathcal{T}^{\gamma}(A)) = \begin{cases} (1,0) & \text{if } A = 0_\sim, 1_\sim, \\ (\frac{1}{2}, \frac{1}{5}) & \text{if } A = A_1, \\ (0,1) & \text{otherwise}; \end{cases}$$

and

$$\mathcal{U}^{\mu\gamma}(A) = (\mathcal{U}^{\mu}(A), \mathcal{U}^{\gamma}(A)) = \begin{cases} (1,0) & \text{if } A = 0_\sim, 1_\sim, \\ (\frac{1}{3}, \frac{1}{4}) & \text{if } A = A_2, \\ (0,1) & \text{otherwise}. \end{cases}$$

Then clearly $(X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ is a double bitopological space on X. Since

$$\begin{aligned} \mathcal{U}^{\mu\gamma}\text{-}cl(\mathcal{T}^{\mu\gamma}\text{-}int(A_3, \frac{1}{2}, \frac{1}{5}), \frac{1}{3}, \frac{1}{4}) &= \mathcal{U}^{\mu\gamma}\text{-}cl(A_1, \frac{1}{3}, \frac{1}{4}) \\ &= 1_\sim \\ &\supseteq A_3, \end{aligned}$$

the intuitionistic fuzzy set A_3 is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(\frac{1}{2}, \frac{1}{5})(\frac{1}{3}, \frac{1}{4})$-semiopen set which is not a $\mathcal{T}^{\mu\gamma}$-fuzzy $(\frac{1}{2}, \frac{1}{5})$-open set. Also, A_3^c is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(\frac{1}{2}, \frac{1}{5})(\frac{1}{3}, \frac{1}{4})$-semiclosed set which is not a $\mathcal{T}^{\mu\gamma}$-fuzzy $(\frac{1}{2}, \frac{1}{5})$-closed set. Since

$$\mathcal{T}^{\mu\gamma}\text{-}cl(\mathcal{U}^{\mu\gamma}\text{-}int(A_4, \frac{1}{3}, \frac{1}{4}), \frac{1}{2}, \frac{1}{5}) = \mathcal{T}^{\mu\gamma}\text{-}cl(A_2, \frac{1}{2}, \frac{1}{5}) = 1_\sim \supseteq A_4,$$

the intuitionistic fuzzy set A_4 is a $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(\frac{1}{3},\frac{1}{4})(\frac{1}{2},\frac{1}{5})$-semiopen set which is not a $\mathcal{U}^{\mu\gamma}$-fuzzy $(\frac{1}{3},\frac{1}{4})$-open set. Also, A_4^c is a $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(\frac{1}{3},\frac{1}{4})(\frac{1}{2},\frac{1}{5})$-semiclosed set which is not a $\mathcal{U}^{\mu\gamma}$-fuzzy $(\frac{1}{3},\frac{1}{4})$-closed set.

THEOREM 3.6. *Let $(X,\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$ be a double bitopological space and $(r,s),(u,v)\in I\otimes I$.*

(1) *If $\{A_k\}$ is a family of $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-semiopen sets of X, then $\bigcup A_k$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-semiopen set.*
(2) *If $\{A_k\}$ is a family of $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-semiopen sets of X, then $\bigcup A_k$ is a $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-semiopen set.*
(3) *If $\{A_k\}$ is a family of $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-semiclosed sets of X, then $\bigcap A_k$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-semiclosed set.*
(4) *If $\{A_k\}$ is a family of $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-semiclosed sets of X, then $\bigcap A_k$ is a $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-semiclosed set.*

Proof. (1) Let $\{A_k\}$ be a collection of $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-semiopen sets. Then for each k, there is a $\mathcal{T}^{\mu\gamma}$-fuzzy (r,s)-open set B_k such that $B_k\subseteq A_k\subseteq\mathcal{U}^{\mu\gamma}\text{-cl}(B_k,u,v)$. Since B_k is $\mathcal{T}^{\mu\gamma}$-fuzzy (r,s)-open, $\mathcal{T}^{\mu}(B_k)\geq r$ and $\mathcal{T}^{\gamma}(B_k)\leq s$ for each k. So $\mathcal{T}^{\mu}(\bigcup B_k)\geq\bigwedge\mathcal{T}^{\mu}(B_k)\geq r$ and $\mathcal{T}^{\gamma}(\bigcup B_k)\leq\bigvee\mathcal{T}^{\gamma}(B_k)\leq s$. Hence $\bigcup B_k$ is a $\mathcal{T}^{\mu\gamma}$-fuzzy (r,s)-open set. Also, we have

$$\begin{aligned}\bigcup B_k\subseteq\bigcup A_k&\subseteq\bigcup\mathcal{U}^{\mu\gamma}\text{-cl}(B_k,u,v)\\&\subseteq\mathcal{U}^{\mu\gamma}\text{-cl}(\bigcup B_k,u,v).\end{aligned}$$

Hence $\bigcup B_k$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-semiopen set.

(2) Let $\{A_k\}$ be a collection of $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-semiopen sets. Then for each k, there is a $\mathcal{U}^{\mu\gamma}$-fuzzy (u,v)-open set B_k such that $B_k\subseteq A_k\subseteq\mathcal{T}^{\mu\gamma}\text{-cl}(B_k,r,s)$. Since B_k is $\mathcal{U}^{\mu\gamma}$-fuzzy (u,v)-open, $\mathcal{U}^{\mu}(B_k)\geq u$ and $\mathcal{U}^{\gamma}(B_k)\leq v$ for each k. So $\mathcal{U}^{\mu}(\bigcup B_k)\geq\bigwedge\mathcal{U}^{\mu}(B_k)\geq u$ and $\mathcal{U}^{\gamma}(\bigcup B_k)\leq\bigvee\mathcal{U}^{\gamma}(B_k)\leq v$. Hence $\bigcup B_k$ is a $\mathcal{U}^{\mu\gamma}$-fuzzy (u,v)-open set. Also, we have

$$\begin{aligned}\bigcup B_k\subseteq\bigcup A_k&\subseteq\bigcup\mathcal{T}^{\mu\gamma}\text{-cl}(B_k,r,s)\\&\subseteq\mathcal{T}^{\mu\gamma}\text{-cl}(\bigcup B_k,r,s).\end{aligned}$$

Hence $\bigcup B_k$ is a $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-semiopen set.

(3) It follows from (1) using Theorem 3.2.

(4) It follows from (2) using Corollary 3.3 $\square$

Let $f : (X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma}) \to (Y, \mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})$ be a mapping from a double bitopological space X to a double bitopological space Y and $(r,s),(u,v) \in I \otimes I$. Then f is called a *double pairwise $(r,s)(u,v)$-continuous ($(r,s)(u,v)$-open and $(r,s)(u,v)$-closed*, respectively) mapping if the induced mapping $f : (X, \mathcal{T}^{\mu\gamma}) \to (Y, \mathcal{V}^{\mu\gamma})$ is fuzzy (r,s)-continuous ((r,s)-open and (r,s)-closed, respectively) and the induced mapping $f : (X, \mathcal{U}^{\mu\gamma}) \to (Y, \mathcal{W}^{\mu\gamma})$ is fuzzy (u,v)-continuous ((u,v)-open and (u,v)-closed, respectively).

Definition 3.7. *Let $f : (X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma}) \to (Y, \mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})$ be a mapping from a double bitopological space X to a double bitopological space Y and $(r,s),(u,v) \in I \otimes I$. Then f is called*

(1) *double pairwise $(r,s)(u,v)$-semicontinuous if $f^{-1}(A)$ is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-semiopen set of X for each $\mathcal{V}^{\mu\gamma}$-fuzzy (r,s)-open set A of Y and $f^{-1}(B)$ is a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-semiopen set of X for each $\mathcal{W}^{\mu\gamma}$-fuzzy (u,v)-open set B of Y,*

(2) *double pairwise $(r,s)(u,v)$-semiopen if $f(C)$ is a $(\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})$-double $(r,s)(u,v)$-semiopen set of Y for each $\mathcal{T}^{\mu\gamma}$-fuzzy (r,s)-open set C of X and $f(D)$ is a $(\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})$-double $(u,v)(r,s)$-semiopen set of Y for each $\mathcal{U}^{\mu\gamma}$-fuzzy (u,v)-open set D of X,*

(3) *double pairwise $(r,s)(u,v)$-semiclosed if $f(C)$ is a $(\mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})$-double $(r,s)(u,v)$-semiclosed set of Y for each $\mathcal{T}^{\mu\gamma}$-fuzzy (r,s)-closed set C of X and $f(D)$ is a $(\mathcal{W}^{\mu\gamma}, \mathcal{V}^{\mu\gamma})$-double $(u,v)(r,s)$-semiclosed set of Y for each $\mathcal{U}^{\mu\gamma}$-fuzzy (u,v)-closed set D of X.*

Theorem 3.8. *Let $f : (X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma}) \to (Y, \mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})$ be a mapping and $(r,s),(u,v) \in I \otimes I$. Then the following statements are equivalent:*

(1) *f is a double pairwise $(r,s)(u,v)$-semicontinuous mapping.*

(2) *$f^{-1}(A)$ is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-semiclosed set of X for each $\mathcal{V}^{\mu\gamma}$-fuzzy (r,s)-closed set A of Y and $f^{-1}(B)$ is a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-semiclosed set of X for each $\mathcal{W}^{\mu\gamma}$-fuzzy (u,v)-closed set B of Y.*

(3) *$\mathcal{U}^{\mu\gamma}$-int$(\mathcal{T}^{\mu\gamma}$-cl$(f^{-1}(A), r, s), u, v) \subseteq f^{-1}(\mathcal{V}^{\mu\gamma}$-cl$(A, r, s))$ and $\mathcal{T}^{\mu\gamma}$-int$(\mathcal{U}^{\mu\gamma}$-cl$(f^{-1}(A), u, v), r, s) \subseteq f^{-1}(\mathcal{W}^{\mu\gamma}$-cl$(A, u, v))$ for each intuitionistic fuzzy set A of Y.*

(4) *$f(\mathcal{U}^{\mu\gamma}$-int$(\mathcal{T}^{\mu\gamma}$-cl$(C, r, s), u, v) \subseteq \mathcal{V}^{\mu\gamma}$-cl$(f(C), r, s)$ and $f(\mathcal{T}^{\mu\gamma}$-int$(\mathcal{U}^{\mu\gamma}$-cl$(C, u, v), r, s) \subseteq \mathcal{W}^{\mu\gamma}$-cl$(f(C), u, v)$ for each intuitionistic fuzzy set C of X.*

Proof. (1) $\Rightarrow$ (2) Let A be any $\mathcal{V}^{\mu\gamma}$-fuzzy (r,s)-closed set and B any $\mathcal{W}^{\mu\gamma}$-fuzzy (u,v)-closed set of Y. Then A^c is a $\mathcal{V}^{\mu\gamma}$-fuzzy (r,s)-open set and B^c is a $\mathcal{W}^{\mu\gamma}$-fuzzy (u,v)-open set of Y. Since f is double pairwise

$(r,s)(u,v)$-semicontinuous, $f^{-1}(A^c)$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-semiopen set and $f^{-1}(B^c)$ is a $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-semiopen set of X. But $f^{-1}(A^c)=f^{-1}(A)^c$ and $f^{-1}(B^c)=f^{-1}(B)^c$. By Theorem 3.2 and Corollary 3.3, $f^{-1}(A)$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-semiclosed set and $f^{-1}(B)$ is a $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-semiclosed set of X.

(2) $\Rightarrow$ (1) Let A be any $\mathcal{V}^{\mu\gamma}$-fuzzy (r,s)-open set and B any $\mathcal{W}^{\mu\gamma}$-fuzzy (u,v)-open set of Y. Then A^c is a $\mathcal{V}^{\mu\gamma}$-fuzzy (r,s)-closed set and B^c is a $\mathcal{W}^{\mu\gamma}$-fuzzy (u,v)-closed set of Y. By (2), $f^{-1}(A^c)$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-semiclosed set and $f^{-1}(B^c)$ is a $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-semiclosed set of X. But $f^{-1}(A^c)=f^{-1}(A)^c$ and $f^{-1}(B^c)=f^{-1}(B)^c$. By Theorem 3.2 and Corollary 3.3, $f^{-1}(A)$ is $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-semiopen and $f^{-1}(B)$ is $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-semiopen of X. Thus f is a double pairwise $(r,s)(u,v)$-semicontinuous mapping.

(2) $\Rightarrow$ (3) Let A be any intuitionistic fuzzy set of Y. Then $\mathcal{V}^{\mu\gamma}$-cl(A,r,s) is a $\mathcal{V}^{\mu\gamma}$-fuzzy (r,s)-closed set and $\mathcal{W}^{\mu\gamma}$-cl(A,u,v) is a $\mathcal{W}^{\mu\gamma}$-fuzzy (u,v)-closed set of Y. By (2), $f^{-1}(\mathcal{V}^{\mu\gamma}\text{-cl}(A,r,s))$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-semiclosed set and $f^{-1}(\mathcal{W}^{\mu\gamma}\text{-cl}(A,u,v))$ is a $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-semiclosed set of X. By Theorem 3.2 and Corollary 3.3,

$$\begin{aligned}
&f^{-1}(\mathcal{V}^{\mu\gamma}\text{-cl}(A,r,s))\\
&\qquad\supseteq \mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(f^{-1}(\mathcal{V}^{\mu\gamma}\text{-cl}(A,r,s)),r,s),u,v)\\
&\qquad\supseteq \mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(f^{-1}(A),r,s),u,v)
\end{aligned}$$

and

$$\begin{aligned}
&f^{-1}(\mathcal{W}^{\mu\gamma}\text{-cl}(A,u,v))\\
&\qquad\supseteq \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(f^{-1}(\mathcal{W}^{\mu\gamma}\text{-cl}(A,u,v)),u,v),r,s)\\
&\qquad\supseteq \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(f^{-1}(A),u,v),r,s).
\end{aligned}$$

(3) $\Rightarrow$ (4) Let C be an intuitionistic fuzzy set of X. Then $f(C)$ is an intuitionistic fuzzy set of Y. By (3),

$$\begin{aligned}
f^{-1}(\mathcal{V}^{\mu\gamma}\text{-cl}(f(C),r,s)) &\supseteq \mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(f^{-1}f(C),r,s),u,v)\\
&\supseteq \mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(C,r,s),u,v)
\end{aligned}$$

and

$$\begin{aligned}
f^{-1}(\mathcal{W}^{\mu\gamma}\text{-cl}(f(C),u,v)) &\supseteq \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(f^{-1}f(C),u,v),r,s)\\
&\supseteq \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(C,u,v),r,s).
\end{aligned}$$

Thus we have

$$\begin{aligned}\mathcal{V}^{\mu\gamma}\text{-cl}(f(C),r,s) &\supseteq ff^{-1}(\mathcal{V}^{\mu\gamma}\text{-cl}(f(C),r,s))\\ &\supseteq f(\mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(C,r,s),u,v))\end{aligned}$$

and

$$\begin{aligned}\mathcal{W}^{\mu\gamma}\text{-cl}(f(C),u,v) &\supseteq ff^{-1}(\mathcal{W}^{\mu\gamma}\text{-cl}(f(C),u,v))\\ &\supseteq f(\mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(C,u,v),r,s)).\end{aligned}$$

(4) $\Rightarrow$ (2) Let A be any $\mathcal{V}^{\mu\gamma}$-fuzzy (r,s)-closed set and B any $\mathcal{W}^{\mu\gamma}$-fuzzy (u,v)-closed set of Y. Then $f^{-1}(A)$ and $f^{-1}(B)$ are intuitionistic fuzzy sets of X. By (4),

$$\begin{aligned}&f(\mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(f^{-1}(A),r,s),u,v))\\ &\qquad\subseteq \mathcal{V}^{\mu\gamma}\text{-cl}(ff^{-1}(A),r,s)\\ &\qquad\subseteq \mathcal{V}^{\mu\gamma}\text{-cl}(A,r,s)\\ &\qquad= A\end{aligned}$$

and

$$\begin{aligned}&f(\mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(f^{-1}(B),u,v),r,s))\\ &\qquad\subseteq \mathcal{W}^{\mu\gamma}\text{-cl}(ff^{-1}(B),u,v)\\ &\qquad\subseteq \mathcal{W}^{\mu\gamma}\text{-cl}(B,u,v)\\ &\qquad= B.\end{aligned}$$

So we have

$$\begin{aligned}&\mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(f^{-1}(A),r,s),u,v)\\ &\qquad\subseteq f^{-1}f(\mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(f^{-1}(A),r,s),u,v)\\ &\qquad\subseteq f^{-1}(A)\end{aligned}$$

and

$$\begin{aligned}&\mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(f^{-1}(B),u,v),r,s)\\ &\qquad\subseteq f^{-1}f(\mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(f^{-1}(B),u,v),r,s)\\ &\qquad\subseteq f^{-1}(B).\end{aligned}$$

Thus $f^{-1}(A)$ is a $(\mathcal{T}^{\mu\gamma},\mathcal{U}^{\mu\gamma})$-double $(r,s)(u,v)$-semiclosed set and $f^{-1}(B)$ is a $(\mathcal{U}^{\mu\gamma},\mathcal{T}^{\mu\gamma})$-double $(u,v)(r,s)$-semiclosed set of X. $\square$

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